Chapter 4 Trigonometric Functions

Section 4.3 Right Angle Trigonometry

Section Objectives: Students will know how to use the fundamental trigonometric identities.

I. The Six Trigonometric Functions (pp. 277–279)  Pace: 15 minutes
- Define the six trigonometric functions, sine, cosine, tangent, cotangent, secant, and cosecant, as follows. Let \( \theta \) be an acute angle of a right triangle. Then

\[
\begin{align*}
\sin \theta &= \frac{\text{opp}}{\text{hyp}} & \cos \theta &= \frac{\text{adj}}{\text{hyp}} & \tan \theta &= \frac{\text{opp}}{\text{adj}} \\
\csc \theta &= \frac{\text{hyp}}{\text{opp}} & \sec \theta &= \frac{\text{hyp}}{\text{adj}} & \cot \theta &= \frac{\text{adj}}{\text{opp}}
\end{align*}
\]

where opp = the length of the side opposite \( \theta \)
adj = the length of the side adjacent to \( \theta \)
hyp = the length of the hypotenuse

Tip: Emphasize that we are shortening the function notation; that is, \( \sin \theta \) is really \( \sin (\theta) \).

- Draw a 45°-45°-90° triangle, labeling both legs as having length 1. Use the Pythagorean Theorem to determine that the length of the hypotenuse is \( \sqrt{2} \).

This leads to
\[
\begin{align*}
\sin 45^\circ &= \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} & \cos 45^\circ &= \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} & \tan 45^\circ &= \frac{1}{1} = 1
\end{align*}
\]

- Now draw a 30°-60°-90° triangle. Remind the students that the length of the hypotenuse of such a triangle is twice the length of the shorter leg. Labels these sides with lengths 2 and 1, respectively. Use the Pythagorean Theorem to determine that the length of the longer leg is \( \sqrt{3} \). This leads to
\[
\begin{align*}
\sin 30^\circ &= \frac{1}{2} & \cos 30^\circ &= \frac{\sqrt{3}}{2} & \tan 30^\circ &= \frac{\sqrt{3}}{3}
\end{align*}
\]
\[
\begin{align*}
\sin 60^\circ &= \frac{\sqrt{3}}{2} & \cos 60^\circ &= \frac{1}{2} & \tan 60^\circ &= \sqrt{3}
\end{align*}
\]

Tip: Since the above nine values are encountered so frequently, students should memorize them.

- Draw attention to the cofunction relationships implied above. Then state
\[
\begin{align*}
\sin (90^\circ - \theta) &= \cos \theta & \cos (90^\circ - \theta) &= \sin \theta \\
\tan (90^\circ - \theta) &= \cot \theta & \cot (90^\circ - \theta) &= \tan \theta \\
\sec (90^\circ - \theta) &= \csc \theta & \csc (90^\circ - \theta) &= \sec \theta
\end{align*}
\]

II. Trigonometric Identities (pp. 280–281)  Pace: 15 minutes
- State the fundamental trigonometric identities in three stages. First, from the definitions, we have the reciprocal identities

\[
\begin{align*}
\sin \theta &= \frac{1}{\csc \theta} & \cos \theta &= \frac{1}{\sec \theta} & \tan \theta &= \frac{1}{\cot \theta} \\
\csc \theta &= \frac{1}{\sin \theta} & \sec \theta &= \frac{1}{\cos \theta} & \cot \theta &= \frac{1}{\tan \theta}
\end{align*}
\]
Second, from the definitions and the reciprocal identities, we have the **quotient identities**
\[
\tan \theta = \frac{\sin \theta}{\cos \theta} \quad \text{cot} \theta = \frac{\cos \theta}{\sin \theta}
\]

Last, from the Pythagorean Theorem, \((\text{opp})^2 + (\text{adj})^2 = (\text{hyp})^2\), and dividing both sides of the equation by \((\text{hyp})^2\), we have the **Pythagorean identities**
\[
\sin^2 \theta + \cos^2 \theta = 1 \quad \tan^2 \theta + 1 = \sec^2 \theta \quad 1 + \cot^2 \theta = \csc^2 \theta.
\]

**Example 1.** If \(\theta\) is an acute angle such that \(\cos \theta = 0.3\), then find the following.

a) \(\sin \theta\)
\[
\sin^2 \theta + \cos^2 \theta = 1 \\
\sin^2 \theta + 0.3^2 = 1 \\
\sin^2 \theta = 1 - 0.3^2 = 0.91 \\
\sin \theta = \sqrt{0.91} \approx \frac{91}{10}
\]

b) \(\tan \theta\)
\[
\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{\sqrt{0.91}}{0.3} = \frac{\sqrt{91}}{3}
\]

c) \(\cot \theta\)
\[
\cot \theta = \frac{1}{\tan \theta} = \frac{0.3}{\sqrt{0.91}} = \frac{3\sqrt{91}}{91}
\]

d) \(\sec \theta\)
\[
\sec \theta = \frac{1}{\cos \theta} = \frac{1}{0.3} = \frac{10}{3}
\]

e) \(\csc \theta\)
\[
\csc \theta = \frac{1}{\sin \theta} = \frac{10}{\sqrt{91}} = \frac{10\sqrt{91}}{91}
\]

**III. Evaluating Trigonometric Functions with a Calculator** (p. 281)

Pace: 5 minutes

**Tip:** In trigonometry, an overwhelming number of incorrect answers come from the calculator being in the wrong mode.

**Example 2.** Evaluate the following by using a calculator:
\(\cos 15.3^\circ\).

After the calculator is in degree mode, enter
\(\text{COS} 15.3 \text{ ENTER}\), \(\cos 15.3^\circ \approx .9646\)

**IV. Applications Involving Right Triangles** (pp. 281–283) Pace: 10 minutes

**Example 3.** If the sun is \(30^\circ\) up from the horizon and shining on a tree forming a 50-foot shadow, how tall is the tree?
\[
\frac{h}{50} = \tan 30^\circ \Rightarrow h = 50 \cdot \frac{\sqrt{3}}{3} \approx 28.87 \text{ feet}.
\]

**Example 4.** If a rope tied to the top of a flagpole is 35 feet long, then what angle is formed by the rope and the ground when the rope is pulled to the ground, 25 feet from the base of the pole?
\(\cos \theta = \frac{25}{35} \Rightarrow \theta \approx 44.42^\circ\).